# 3D LUT interpolation 

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## LUT: Look Up Table

- A faster way to implement a function between two discrete finite sets. $A=\left\{0,1,2,3, \ldots, 2^{N}-1\right\}, f: A \rightarrow A$
- A value comes in and the corresponding output value comes out.
- For a 10bit video, the code book has $2^{10}=1024$ entries for each primary (red, green, blue).

Example:

| 0 | 0 |
| :---: | :---: |
| 1 | 0 |
| 2 | 15 |
| 3 | 125 |
| 4 | 14 |
| $\vdots$ | $\vdots$ |
| 1023 | 1020 |

LUT: Look Up Table - 1D

- Such a LUT is referred to as a 1 dimensional LUT (or 1D LUT)
- One 1D LUT can be used to alter the luminance of the image (tone mapping)
- $3 \times 1 \mathrm{~L}$ LUT can be used to alter each color primary separately but can't alter a particular color without affecting others $(r, g, b)$
- A 10bit video signal is composed of a pair of 3 values between 0 and $2^{10}-1=1023$ for each transported pixel.
- Example:
- $(0,0,0)$ - black
- (1023, 1023, 1023) - white
- (1023, 0, 0)-red
- $(0,1023,0)$ - green
- $(0,0,1023)$ - blue
- $(1023,1023,0)$ - yellow


## LUT: Look Up Table - 3D

- A 3D LUT is a LUT containing entries for each possible $(r, g, b)$ triplets.
- Problem: For a 10 -bit video signal, this table would have $\left(2^{10}\right)^{3}=1,073,741,824$ entries each containing a 30 -bit value $(10$ bits per channel $)=32,212,254,720$ bits $=$ 4.02653184 Gigabyte.
- The system (hardware or software) would have to parse through a 4.03 Gigabyte memory for every pixel.
- In 4 K (UHD: $3,840 \times 2,160$ ) at 30 fps , that is $8,294,400$ pixels 30 times per second or $248,832,000$ scans of that Look Up Table per second!

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- We need to find another way

LUT: Look Up Table

- Solution: storing a sparse 3D LUT and use interpolation to recover values.


## LUT: Look Up Table

 PROS:- Very light file.
- Typically $33^{3}=35,937$ entries
- intuitive and easy to use.

CONS:

- Error due to interpolation
- All the colors are treated the same way
- Hardware often impose table to be be RGB

3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation
Notation: Upper case for mesh points. lower case for points non on the grid.
$V(r, g, b)$ is the value at the point with coordinate $(r, g, b)$.
We start by calculating the distance between each node per coordinate:

$$
\begin{aligned}
\Delta_{r} & =\frac{r-R_{0}}{R_{1}-R_{0}} \\
\Delta_{g} & =\frac{g-G_{0}}{G_{1}-G_{0}} \\
\Delta_{b} & =\frac{b-B_{0}}{B_{1}-B_{0}}
\end{aligned}
$$

This is simply the proportion of each before and after mesh point there is in the point $x$.

3D LUT interpolation: Trilinear interpolation
We start with red (the result is independent of the order) and calculate the value at each 4 points by doing the weighted average:

$$
\begin{aligned}
& V\left(r, G_{0}, B_{0}\right)=V\left(R_{0}, G_{0}, B_{0}\right)\left(1-\Delta_{r}\right)+V\left(R_{1}, G_{0}, B_{0}\right) \Delta_{r} \\
& V\left(r, G_{0}, B_{1}\right)=V\left(R_{0}, G_{0}, B_{1}\right)\left(1-\Delta_{r}\right)+V\left(R_{1}, G_{0}, B_{1}\right) \Delta_{r} \\
& V\left(r, G_{1}, B_{0}\right)=V\left(R_{0}, G_{1}, B_{0}\right)\left(1-\Delta_{r}\right)+V\left(R_{1}, G_{1}, B_{0}\right) \Delta_{r} \\
& V\left(r, G_{1}, B_{1}\right)=V\left(R_{0}, G_{1}, B_{1}\right)\left(1-\Delta_{r}\right)+V\left(R_{1}, G_{1}, B_{1}\right) \Delta_{r}
\end{aligned}
$$

3D LUT interpolation: Trilinear interpolation
Now that we have computed those values, we move on to the green channel:

$$
\begin{aligned}
& V\left(r, g, B_{0}\right)=V\left(r, G_{0}, B_{0}\right)\left(1-\Delta_{g}\right)+V\left(r, G_{1}, B_{0}\right) \Delta_{g} \\
& V\left(r, g, B_{1}\right)=V\left(r, G_{0}, B_{1}\right)\left(1-\Delta_{g}\right)+V\left(r, G_{1}, B_{1}\right) \Delta_{g}
\end{aligned}
$$

Finally we can compute the value at the point ( $r, g, b$ ) interpolating the blue channel:

$$
V(r, g, b)=V\left(r, g, B_{0}\right)\left(1-\Delta_{b}\right)+V\left(r, g, B_{1}\right) \Delta_{b}
$$

3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation


3D LUT interpolation: Trilinear interpolation
The general expression for the trilinear interpolation can be expressed as

$$
\left.\begin{array}{rl}
V(r, g, b)= & c_{0} \tag{1}
\end{array}+c_{1} \Delta_{b}+c_{2} \Delta_{r}+c_{3} \Delta_{g}+c_{4} \Delta_{b} \Delta_{r}\right)
$$

with:

$$
\begin{aligned}
& c_{0}=V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{1}=V\left(R_{0}, G_{0}, B_{1}\right)-V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{2}=V\left(R_{1}, G_{0}, B_{0}\right)-V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{3}=V\left(R_{0}, G_{1}, B_{0}\right)-V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{4}=V\left(R_{1}, G_{0}, B_{1}\right)-V\left(R_{1}, G_{0}, B_{0}\right)-V\left(R_{0}, G_{0}, B_{1}\right)+V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{5}=V\left(R_{1}, G_{1}, B_{0}\right)-V\left(R_{0}, G_{1}, B_{0}\right)-V\left(R_{1}, G_{0}, B_{0}\right)+V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{6}=V\left(R_{0}, G_{1}, B_{1}\right)-V\left(R_{0}, G_{1}, B_{0}\right)-V\left(R_{0}, G_{0}, B_{1}\right)+V\left(R_{0}, G_{0}, B_{0}\right) \\
& c_{7}=V\left(R_{1}, G_{1}, B_{1}\right)-V\left(R_{1}, G_{1}, B_{0}\right)-V\left(R_{0}, G_{1}, B_{1}\right)-V\left(R_{1}, G_{0}, B_{1}\right) \\
& +V\left(R_{0}, G_{0}, B_{1}\right)+V\left(R_{0}, G_{1}, B_{0}\right)+V\left(R_{1}, G_{0}, B_{0}\right)-V\left(R_{0}, G_{0}, B_{0}\right)
\end{aligned}
$$

3D LUT interpolation: Trilinear interpolation
Expressed in matrix form:

$$
\begin{gathered}
\boldsymbol{C}=\left[\begin{array}{llllllll}
c_{0} & c_{1} & c_{2} & c_{3} & c_{4} & c_{5} & c_{6} & c_{7}
\end{array}\right]^{T} \\
\Delta=\left[\begin{array}{llllllll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{b} \Delta_{r} & \Delta_{r} \Delta_{g} & \Delta_{g} \Delta_{b} & \Delta_{r} \Delta_{g} \Delta_{b}
\end{array}\right]^{T} \\
V(r, g, b)=C^{T} \Delta
\end{gathered}
$$

3D LUT interpolation: Trilinear interpolation
Expressed in matrix form:

$$
\left[\begin{array}{l}
c_{0} \\
c_{1} \\
c_{2} \\
c_{3} \\
c_{4} \\
c_{5} \\
c_{6} \\
c_{7}
\end{array}\right]=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\
-1 & 1 & 1 & -1 & 1 & -1 & -1 & 1
\end{array}\right]\left[\begin{array}{l}
V\left(R_{0}, G_{0}, B_{0}\right) \\
V\left(R_{0}, G_{1}, B_{0}\right) \\
V\left(R_{1}, G_{0}, B_{0}\right) \\
V\left(R_{1}, G_{1}, B_{0}\right) \\
V\left(R_{0}, G_{0}, B_{1}\right) \\
V\left(R_{0}, G_{1}, B_{1}\right) \\
V\left(R_{1}, G_{0}, B_{1}\right) \\
V\left(R_{1}, G_{1}, B_{1}\right)
\end{array}\right]
$$

The expression above can be written as:
$\boldsymbol{C}=\boldsymbol{A} \boldsymbol{V}$
And the trilinear interpolation as:
$V(r, g, b)=\boldsymbol{C}^{\top} \boldsymbol{\Delta}=\boldsymbol{V}^{T} \boldsymbol{A}^{T} \boldsymbol{\Delta}$

3D LUT interpolation: Trilinear interpolation
And the trilinear interpolation can be written as:
$V(r, g, b)=\boldsymbol{C}^{\top} \boldsymbol{\Delta}=\boldsymbol{V}^{T} \boldsymbol{A}^{T} \boldsymbol{\Delta}$
Note that the term $\boldsymbol{V}^{T} \boldsymbol{A}^{T}$ doesn't depend on the variable ( $r, g, b$ ) and thus can be computed in advance. So, each sub-cube can have the values of the vector $\boldsymbol{C}$ already stored in memory. Therefore the algorithm can be summarize as:

- Find the sub-cube the point $(r, g, b)$ is located in.
- Select the vector $\boldsymbol{C}$ corresponding to that sub-cube.
- Compute $\Delta_{r}, \Delta_{g}, \Delta_{b}$
- Return $V(r, g, b)=\boldsymbol{C}^{T} \boldsymbol{\Delta}$

3D LUT interpolation: other interpolations
One idea to speed-up the computation (and hopefully increase its accuracy) is to use a smaller sub-section of the cube. There are only three ways of slicing a cube into multiple 3D structures with equal number of vertices:

- Prisms (each having six vertices)
- Pyramids (each having five vertices)
- Tetrahedrons (each having four vertices)

3D LUT interpolation: Prism interpolation


The algorithm chooses the prism based on the cases:

- if $\Delta_{b}>\Delta_{r}$, we choose the left prism ( $p 1$ ).
- if $\Delta_{b}<\Delta_{r}$, we choose the right prism ( $p 2$ ).
- if $\Delta_{b}=\Delta_{r}$, we choose either prism.

3D LUT interpolation: Prism interpolation
Triangular interpolation:
$V\left(R_{0}, G_{0}, B_{0}\right)$


3D LUT interpolation: Prism interpolation
Triangular interpolation:
$V\left(R_{0}, G_{0}, B_{0}\right)$

$V(r, g, b)=$
$V\left(R_{0}, G_{0}, B_{0}\right) \frac{S_{\text {ono }}}{S_{\text {tot }}}+V\left(R_{0}, G_{0}, B_{1}\right) \frac{S_{\text {ono }}}{S_{\text {tot }}}+V\left(R_{1}, G_{0}, B_{0}\right) \frac{S_{\text {100 }}}{S_{\text {tot }}}$

Triangular interpolation:


Surface is given by the shoelace formula:

$$
S_{000}=\frac{1}{2}\left|\operatorname{det}\left[\begin{array}{ccc}
R_{0} & R_{1} & r \\
B_{1} & B_{0} & b \\
1 & 1 & 1
\end{array}\right]\right|=\frac{1}{2}\left|R_{0} B_{0}+R 1 b+r B_{1}-R_{0} b-R_{1} B_{1}-r B_{0}\right|
$$

3D LUT interpolation: Prism interpolation


$$
\begin{align*}
V(r, g, b)=(V & \left.\left(R_{0}, G_{0}, B_{0}\right) \frac{S_{000}}{S_{t o t}}+V\left(R_{0}, G_{0}, B_{1}\right) \frac{S_{001}}{S_{t o t}}+V\left(R_{1}, G_{0}, B_{1}\right) \frac{S_{101}}{S_{t o t}}\right)\left(1-\Delta_{g}\right) \\
& +\left(V\left(R_{0}, G_{1}, B_{0}\right) \frac{S_{010}}{S_{t o t}}+V\left(R_{0}, G_{1}, B_{1}\right) \frac{S_{011}}{S_{t o t}}+V\left(R_{1}, G_{1}, B_{1}\right) \frac{S_{111}}{S_{t o t}}\right) \Delta_{g} \tag{2}
\end{align*}
$$

Note that $S_{000}=S_{010}, S_{001}=S_{011}$ and $S_{101}=S_{111}$.

3D LUT interpolation: Prism interpolation
We use the matrix notation and define the solution for both prisms ( $p 1, p 2$ ) using the following vectors:

$$
\begin{gathered}
\boldsymbol{V}=\left[\begin{array}{c}
V\left(R_{0}, G_{0}, B_{0}\right) \\
V\left(R_{0}, G_{1}, B_{0}\right) \\
V\left(R_{1}, G_{0}, B_{0}\right) \\
V\left(R_{1}, G_{1}, B_{0}\right) \\
V\left(R_{0}, G_{0}, B_{1}\right) \\
V\left(R_{0}, G_{1}, B_{1}\right) \\
V\left(R_{1}, G_{0}, B_{1}\right) \\
V\left(R_{1}, G_{1}, B_{1}\right)
\end{array}\right] \\
\boldsymbol{\Delta}_{p}=\left[\begin{array}{llllll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{b} \Delta_{g} & \Delta_{r} \Delta_{g}
\end{array}\right]^{T}
\end{gathered}
$$

3D LUT interpolation: Prism interpolation
And by defining the two following matrices:

$$
\begin{aligned}
& \boldsymbol{B}_{\mathbf{1}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1
\end{array}\right] \\
& \boldsymbol{B}_{\mathbf{2}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0
\end{array}\right]
\end{aligned}
$$

3D LUT interpolation: Prism interpolation
Now we can find the interpolation for each prism as

$$
\begin{aligned}
& V(r, g, b)_{p_{1}}=\boldsymbol{\Delta}_{p}^{T} \boldsymbol{B}_{\mathbf{1}} \boldsymbol{V} \\
& V(r, g, b)_{p_{2}}=\boldsymbol{\Delta}_{p}^{T} \boldsymbol{B}_{\mathbf{2}} \boldsymbol{V}
\end{aligned}
$$

We have the same situation as with the trilinear interpolation as the matrices $\boldsymbol{B}_{1} \boldsymbol{V}$ and $\boldsymbol{B}_{2} \boldsymbol{V}$ can be computed and stored in advance since they don't depend on the coordinates ( $r, g, b$ ) or $\Delta_{r}, \Delta_{g}$ and $\Delta_{b}$. That strategy could speed up the interpolation process.

3D LUT interpolation: Pyramid interpolation



3D LUT interpolation: Pyramid interpolation


3D LUT interpolation: Pyramid interpolation


3D LUT interpolation: Pyramid interpolation


$$
\begin{align*}
V(r, g, b)=V\left(R_{0}, G_{0}, B_{0}\right) \frac{V_{000}}{V_{\text {Prism } 1}} & +V\left(R_{0}, G_{0}, B_{1}\right) \frac{V_{001}}{V_{\text {Prism } 1}} \\
+V\left(R_{1}, G_{0}, B_{1}\right) \frac{V_{101}}{V_{\text {Prism } 1}} & +V\left(R_{0}, G_{1}, B_{1}\right) \frac{V_{011}}{V_{\text {Prism } 1}}  \tag{3}\\
& +V\left(R_{1}, G_{1}, B_{1}\right) \frac{V_{111}}{V_{\text {Prism } 1}}
\end{align*}
$$

Rmk:
$V_{\text {Prism } 1}=V_{000}+V_{001}+V_{101}+V_{011}+V_{111}$

3D LUT interpolation: Pyramid interpolation


The algorithm chooses the pyramid based on the cases:

- if $\Delta_{r}>\Delta_{b}$ and $\Delta_{g}>\Delta_{b}$, then $p=p_{1}$.
- if $\Delta_{b}>\Delta_{r}$ and $\Delta_{g}>\Delta_{r}$, then $p=p_{2}$.
- else $p=p_{3}$.

3D LUT interpolation: Pyramid interpolation
We use the matrix notation and define the solution or the three pyramids $\left(p_{1}, p_{2}, p_{3}\right)$ :

$$
\begin{gathered}
\boldsymbol{V}=\left[\begin{array}{c}
V\left(R_{0}, G_{0}, B_{0}\right) \\
V\left(R_{0}, G_{1}, B_{0}\right) \\
V\left(R_{1}, G_{0}, B_{0}\right) \\
V\left(R_{1}, G_{1}, B_{0}\right) \\
V\left(R_{0}, G_{0}, B_{1}\right) \\
V\left(R_{0}, G_{1}, B_{1}\right) \\
V\left(R_{1}, G_{0}, B_{1}\right) \\
V\left(R_{1}, G_{1}, B_{1}\right)
\end{array}\right] \\
\boldsymbol{\Delta}_{p_{1}}=\left[\begin{array}{lllll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{r} \Delta_{g}
\end{array}\right]^{T} \\
\boldsymbol{\Delta}_{p_{2}}=\left[\begin{array}{lllll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{g} \Delta_{b}
\end{array}\right]^{T} \\
\boldsymbol{\Delta}_{p_{3}}=\left[\begin{array}{lllll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{b} \Delta_{r}
\end{array}\right]^{T}
\end{gathered}
$$

3D LUT interpolation: Pyramid interpolation
And by defining the three following matrices:

$$
\begin{aligned}
& \boldsymbol{C}_{\mathbf{1}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & -1 & 1 & 0 & 0 & 0 & 0
\end{array}\right] \\
& \boldsymbol{C}_{2}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & -1 & 1 & 0 & 0
\end{array}\right] \\
& \boldsymbol{C}_{3}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
1 & 0 & -1 & 0 & -1 & 0 & 1 & 0
\end{array}\right]
\end{aligned}
$$

3D LUT interpolation: Pyramid interpolation
Now we can find the interpolation for each pyramid as

$$
\begin{aligned}
& V(r, g, b)_{p_{1}}=\Delta_{p_{1}}^{T} C_{1} V \\
& V(r, g, b)_{p_{2}}=\Delta_{p_{2}}^{T} C_{2} V \\
& V(r, g, b)_{p_{3}}=\Delta_{p_{3}}^{T} C_{3} V
\end{aligned}
$$

Again, the vectors $\boldsymbol{C}_{1} \boldsymbol{V}, \boldsymbol{C}_{2} \boldsymbol{V}$ and $\boldsymbol{C}_{3} \boldsymbol{V}$ can be computed and stored in advance since they don't depend on the coordinates $(r, g, b)$ or $\Delta_{r}, \Delta_{g}$ and $\Delta_{b}$. That strategy could speed up the interpolation process.

3D LUT interpolation: Tetrahedral interpolation


3D LUT interpolation: Tetrahedral interpolation
The algorithm chooses the tetrahedron based on the cases:

- if $\Delta_{b}>\Delta_{r}>\Delta_{g}$, we chose the first tetrahedron ( $t 1$ ),
- if $\Delta_{b}>\Delta_{g}>\Delta_{r}$, we chose the second tetrahedron ( $t 2$ ),
- if $\Delta_{g}>\Delta_{b}>\Delta_{r}$, we chose the third tetrahedron ( $t 3$ ),
- if $\Delta_{r}>\Delta_{b}>\Delta_{g}$, we chose the fourth tetrahedron ( $t 4$ ),
- if $\Delta_{r}>\Delta_{g}>\Delta_{b}$, we chose the fifth tetrahedron ( $t 5$ ),
- else we chose the sixth tetrahedron ( $t 6$ ).

3D LUT interpolation: Tetrahedral interpolation We use the matrix notation:

$$
\begin{gathered}
\boldsymbol{V}=\left[\begin{array}{c}
V\left(R_{0}, G_{0}, B_{0}\right) \\
V\left(R_{0}, G_{1}, B_{0}\right) \\
V\left(R_{1}, G_{0}, B_{0}\right) \\
V\left(R_{1}, G_{1}, B_{0}\right) \\
V\left(R_{0}, G_{0}, B_{1}\right) \\
V\left(R_{0}, G_{1}, B_{1}\right) \\
V\left(R_{1}, G_{0}, B_{1}\right) \\
V\left(R_{1}, G_{1}, B_{1}\right)
\end{array}\right] \\
\boldsymbol{\Delta}_{t}=\left[\begin{array}{llll}
1 & \Delta_{b} & \Delta_{r} & \Delta_{g}
\end{array}\right]^{T}
\end{gathered}
$$

3D LUT interpolation: Terahedral interpolation
And by defining the six following matrices:

$$
\begin{aligned}
& \boldsymbol{T}_{\mathbf{1}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right] \\
& \boldsymbol{T}_{\mathbf{2}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0
\end{array}\right] \\
& \boldsymbol{T}_{\mathbf{3}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{aligned}
$$

3D LUT interpolation: Terahedral interpolation

$$
\begin{aligned}
& \boldsymbol{T}_{4}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{array}\right] \\
& \boldsymbol{T}_{\mathbf{5}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0
\end{array}\right] \\
& \boldsymbol{T}_{\mathbf{6}}=\left[\begin{array}{cccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{aligned}
$$

3D LUT interpolation: Tetrahedral interpolation
Now we can find the interpolation for each tetrahedron as

$$
\begin{array}{ll}
V(r, g, b)_{t_{1}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{\mathbf{1}} \boldsymbol{V} & V(r, g, b)_{t_{2}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{\mathbf{2}} \boldsymbol{V} \\
V(r, g, b)_{t_{3}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{\mathbf{3}} \boldsymbol{V} & V(r, g, b)_{t_{4}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{4} \boldsymbol{V} \\
V(r, g, b)_{t_{5}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{\mathbf{5}} \boldsymbol{V} & V(r, g, b)_{t_{6}}=\boldsymbol{\Delta}_{t}^{T} \boldsymbol{T}_{\mathbf{6}} \boldsymbol{V}
\end{array}
$$

Again, the vectors $\boldsymbol{T}_{\boldsymbol{i}} \boldsymbol{V}$, for $i=1,2,3,4,5,6$, can be computed and stored in advance since they don't depend on the coordinates $(r, g, b)$ or $\Delta_{r}, \Delta_{g}$ and $\Delta_{b}$. That strategy could speed up the interpolation process.

3D LUT interpolation: comparison of computational cost

|  | Comparisons | Multiplications | Additions | Storage* |
| :--- | :---: | :---: | :---: | :---: |
| Trilinear | 0 | 7 | 7 | 12 |
| Prism | 1 | 5 | 6 | 9 |
| Pyramidal | 2 | 4 | 4 | 5 |
| Tetrahedron | 2.5 | 3 | 3 | 13 |

* pre-computed coefficient stored at each node

3D LUT interpolation: other approaches

Other ways to interpolate the 3D LUT:

- Cellular Regression
- Non-uniform lattice 3D LUT
- Alternative color spaces

3D LUT interpolation: Cellular Regression

A combination of three-dimensional interpolation and cellular regression.
We apply regression to a small lattice cell (versus the whole cube).
Pros:

- No need to find the position of the interpolation within the cube.
- No need for uniform packing: new 3D structures, like hexahedra, can be used.

Cons:

- Higher computational cost
- Why not use a larger 3D LUT then?

3D LUT interpolation: Non-uniform lattice 3D LUT

Pros:

- Perception non-uniformity: We don't see difference between two colors the same depending on where we are in the cube.
- Does not require additional computational cost (most hardware feature a front $3 \times 1$ LUT)
Cons:
- Not 'hardware friendly'

3D LUT interpolation: Non-uniform lattice 3D LUT


Acknowledgment

- C. Poynton, Digital Video and HDTV Algorithm and Interfaces, Morgan Kaufmann, San Francisco, 2003.
- Henry R. Kang, Computational Color Technology, SPIE Press Book, 17 May 2006.

