3D LUT interpolation

JD Vandenberg August 28th, 2019

LUT: Look Up Table

- A faster way to implement a function between two discrete finite sets. $A = \{0, 1, 2, 3, \dots, 2^N 1\}, f : A \rightarrow A$
- ► A value comes in and the corresponding output value comes out.
- For a 10bit video, the code book has $2^{10} = 1024$ entries for each primary (red, green, blue).

Example:

0	0
1	0
2	15
3	125
4	14
:	÷
1023	1020

LUT: Look Up Table - 1D

- Such a LUT is referred to as a 1 dimensional LUT (or 1D LUT)
- One 1D LUT can be used to alter the luminance of the image (tone mapping)
- ▶ 3x 1D LUT can be used to alter each color primary separately but can't alter a particular color without affecting others (r, g, b)
- A 10bit video signal is composed of a pair of 3 values between 0 and $2^{10} 1 = 1023$ for each transported pixel.
- Example:
 - ► (0,0,0) black
 - ► (1023, 1023, 1023) white
 - ► (1023, 0, 0) red
 - ▶ (0,1023,0) green
 - ► (0,0,1023) blue
 - ► (1023, 1023, 0) yellow
 - . . .

LUT: Look Up Table - 3D

- ▶ A 3D LUT is a LUT containing entries for each possible (r, g, b) triplets.
- ▶ Problem: For a 10-bit video signal, this table would have $(2^{10})^3 = 1,073,741,824$ entries each containing a 30-bit value (10 bits per channel) = 32,212,254,720 bits = 4.02653184 Gigabyte.
- ► The system (hardware or software) would have to parse through a 4.03 Gigabyte memory for every pixel.
- ► In 4K (UHD: 3,840x2,160) at 30fps, that is 8,294,400 pixels 30 times per second or 248,832,000 scans of that Look Up Table per second!

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- ► The system (hardware or software) would have to parse through a 4.03 Gigabyte memory for every pixel.
- ► In 4K (UHD: 3,840x2,160) at 30fps, that is 8,294,400 pixels 30 times per second or 248,832,000 scans of that Look Up Table per second!
- ► We need to find another way

LUT: Look Up Table

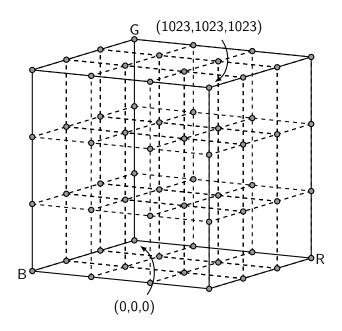
► Solution: storing a sparse 3D LUT and use interpolation to recover values.

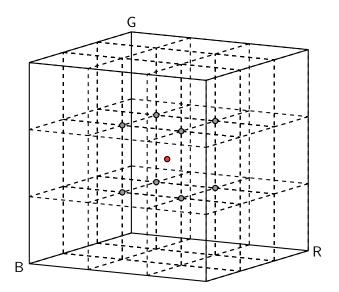
LUT: Look Up Table PROS:

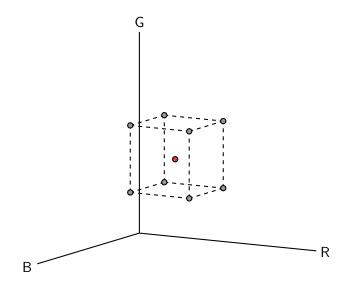
- Very light file.
- ▶ Typically $33^3 = 35,937$ entries
- intuitive and easy to use.

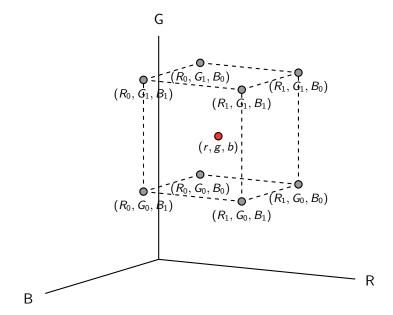
CONS:

- Error due to interpolation
- All the colors are treated the same way
- ► Hardware often impose table to be be RGB









Notation: Upper case for mesh points. lower case for points non on the grid.

V(r,g,b) is the value at the point with coordinate (r,g,b). We start by calculating the distance between each node per coordinate:

$$\Delta_r = \frac{r - R_0}{R_1 - R_0}$$

$$\Delta_g = \frac{g - G_0}{G_1 - G_0}$$

$$\Delta_b = \frac{b - B_0}{B_1 - B_0}$$

This is simply the proportion of each before and after mesh point there is in the point x.

We start with red (the result is independent of the order) and calculate the value at each 4 points by doing the weighted average:

$$V(r, G_0, B_0) = V(R_0, G_0, B_0)(1 - \Delta_r) + V(R_1, G_0, B_0)\Delta_r$$
 $V(r, G_0, B_1) = V(R_0, G_0, B_1)(1 - \Delta_r) + V(R_1, G_0, B_1)\Delta_r$
 $V(r, G_1, B_0) = V(R_0, G_1, B_0)(1 - \Delta_r) + V(R_1, G_1, B_0)\Delta_r$
 $V(r, G_1, B_1) = V(R_0, G_1, B_1)(1 - \Delta_r) + V(R_1, G_1, B_1)\Delta_r$

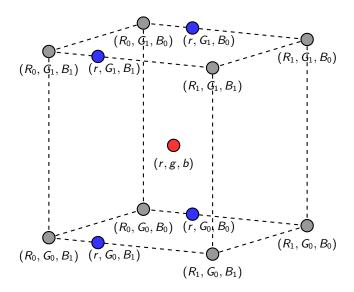
Now that we have computed those values, we move on to the green channel:

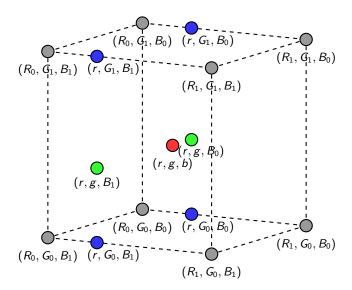
$$V(r, g, B_0) = V(r, G_0, B_0)(1 - \Delta_g) + V(r, G_1, B_0)\Delta_g$$

 $V(r, g, B_1) = V(r, G_0, B_1)(1 - \Delta_g) + V(r, G_1, B_1)\Delta_g$

Finally we can compute the value at the point (r, g, b) interpolating the blue channel:

$$V(r, g, b) = V(r, g, B_0)(1 - \Delta_b) + V(r, g, B_1)\Delta_b$$





The general expression for the trilinear interpolation can be expressed as

$$V(r,g,b) = c_0 + c_1 \Delta_b + c_2 \Delta_r + c_3 \Delta_g + c_4 \Delta_b \Delta_r + c_5 \Delta_r \Delta_g + c_6 \Delta_g \Delta_b + c_7 \Delta_r \Delta_g \Delta_b$$
 (1)

with: $c_0 = V(R_0, G_0, B_0)$ $c_1 = V(R_0, G_0, B_1) - V(R_0, G_0, B_0)$ $c_2 = V(R_1, G_0, B_0) - V(R_0, G_0, B_0)$ $c_3 = V(R_0, G_1, B_0) - V(R_0, G_0, B_0)$ $c_4 = V(R_1, G_0, B_1) - V(R_1, G_0, B_0) - V(R_0, G_0, B_1) + V(R_0, G_0, B_0)$ $c_5 = V(R_1, G_1, B_0) - V(R_0, G_1, B_0) - V(R_1, G_0, B_0) + V(R_0, G_0, B_0)$ $c_6 = V(R_0, G_1, B_1) - V(R_0, G_1, B_0) - V(R_0, G_0, B_1) + V(R_0, G_0, B_0)$ $c_7 = V(R_1, G_1, B_1) - V(R_1, G_1, B_0) - V(R_0, G_1, B_1) - V(R_1, G_0, B_1)$ $+ V(R_0, G_0, B_1) + V(R_0, G_1, B_0) + V(R_1, G_0, B_0) - V(R_0, G_0, B_0)$

Expressed in matrix form:

$$\boldsymbol{C} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \end{bmatrix}^T$$

$$\boldsymbol{\Delta} = \begin{bmatrix} 1 & \Delta_b & \Delta_r & \Delta_g & \Delta_b \Delta_r & \Delta_r \Delta_g & \Delta_g \Delta_b & \Delta_r \Delta_g \Delta_b \end{bmatrix}^T$$

$$V(r,g,b) = \mathbf{C}^T \mathbf{\Delta}$$

Expressed in matrix form:

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} V(R_0, G_0, B_0) \\ V(R_0, G_1, B_0) \\ V(R_1, G_0, B_0) \\ V(R_1, G_0, B_0) \\ V(R_0, G_0, B_1) \\ V(R_0, G_1, B_1) \\ V(R_1, G_0, B_1) \\ V(R_1, G_0, B_1) \end{bmatrix}$$

The expression above can be written as:

$$C = AV$$

And the trilinear interpolation as:

$$V(r,g,b) = \mathbf{C}^T \mathbf{\Delta} = \mathbf{V}^T \mathbf{A}^T \mathbf{\Delta}$$

And the trilinear interpolation can be written as:

$$V(r,g,b) = \mathbf{C}^T \mathbf{\Delta} = \mathbf{V}^T \mathbf{A}^T \mathbf{\Delta}$$

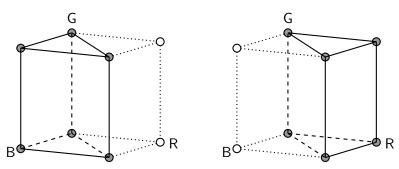
Note that the term $V^T A^T$ doesn't depend on the variable (r, g, b) and thus can be computed in advance. So, each sub-cube can have the values of the vector C already stored in memory. Therefore the algorithm can be summarize as:

- Find the sub-cube the point (r, g, b) is located in.
- ▶ Select the vector **C** corresponding to that sub-cube.
- ► Compute Δ_r , Δ_g , Δ_b

3D LUT interpolation: other interpolations

One idea to speed-up the computation (and hopefully increase its accuracy) is to use a smaller sub-section of the cube. There are only three ways of slicing a cube into multiple 3D structures with equal number of vertices:

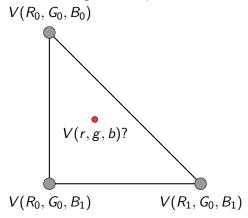
- Prisms (each having six vertices)
- Pyramids (each having five vertices)
- Tetrahedrons (each having four vertices)



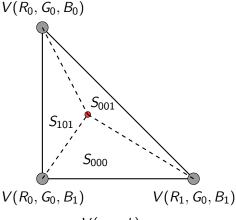
The algorithm chooses the prism based on the cases:

- ▶ if $\Delta_b > \Delta_r$, we choose the left prism (p1).
- ▶ if $\Delta_b < \Delta_r$, we choose the right prism (p2).
- ightharpoonup if $\Delta_b = \Delta_r$, we choose either prism.

Triangular interpolation:



Triangular interpolation:

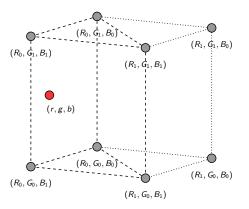


$$V(r,g,b) = V(R_0,G_0,B_0) \frac{S_{000}}{S_{tot}} + V(R_0,G_0,B_1) \frac{S_{001}}{S_{tot}} + V(R_1,G_0,B_0) \frac{S_{100}}{S_{tot}}$$

Triangular interpolation: $V(R_0, G_0, B_0)$ S_{101} , S_{000} $V(R_0, G_0, B_1)$ $V(R_1, G_0, B_1)$

Surface is given by the shoelace formula:

$$S_{000} = \frac{1}{2} |\det \begin{bmatrix} R_0 & R_1 & r \\ B_1 & B_0 & b \\ 1 & 1 & 1 \end{bmatrix}| = \frac{1}{2} |R_0 B_0 + R_1 b + r B_1 - R_0 b - R_1 B_1 - r B_0|$$



$$V(r,g,b) = \left(V(R_0, G_0, B_0) \frac{S_{000}}{S_{tot}} + V(R_0, G_0, B_1) \frac{S_{001}}{S_{tot}} + V(R_1, G_0, B_1) \frac{S_{101}}{S_{tot}}\right) (1 - \Delta_g)$$

$$+ \left(V(R_0, G_1, B_0) \frac{S_{010}}{S_{tot}} + V(R_0, G_1, B_1) \frac{S_{011}}{S_{tot}} + V(R_1, G_1, B_1) \frac{S_{111}}{S_{tot}}\right) \Delta_g$$
(2)

Note that $S_{000} = S_{010}$, $S_{001} = S_{011}$ and $S_{101} = S_{111}$.

We use the matrix notation and define the solution for both prisms (p1, p2) using the following vectors:

$$oldsymbol{V} = egin{bmatrix} V(R_0,G_0,B_0) \ V(R_0,G_1,B_0) \ V(R_1,G_0,B_0) \ V(R_1,G_1,B_0) \ V(R_0,G_0,B_1) \ V(R_0,G_1,B_1) \ V(R_1,G_0,B_1) \ V(R_1,G_0,B_1) \end{bmatrix}$$

$$oldsymbol{\Delta}_{p} = egin{bmatrix} 1 & \Delta_{b} & \Delta_{r} & \Delta_{g} & \Delta_{b}\Delta_{g} & \Delta_{r}\Delta_{g} \end{bmatrix}^{T}$$

And by defining the two following matrices:

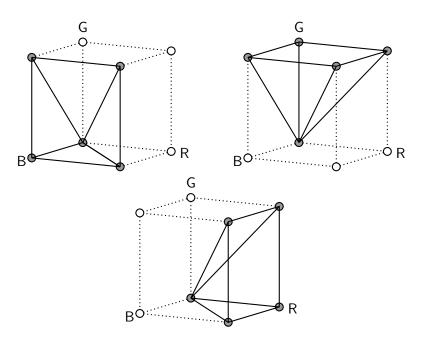
$$m{B_1} = egin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \ 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

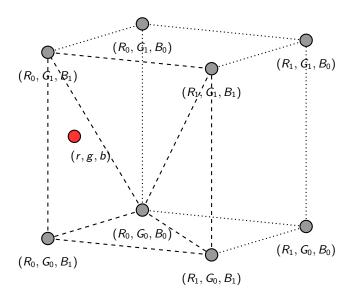
Now we can find the interpolation for each prism as

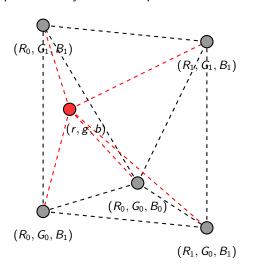
$$V(r,g,b)_{p_1} = \boldsymbol{\Delta}_p^T \boldsymbol{B_1} \boldsymbol{V}$$

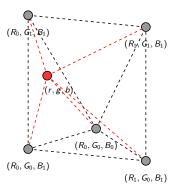
$$V(r,g,b)_{p_2} = \boldsymbol{\Delta}_p^T \boldsymbol{B_2} \boldsymbol{V}$$

We have the same situation as with the trilinear interpolation as the matrices B_1V and B_2V can be computed and stored in advance since they don't depend on the coordinates (r,g,b) or Δ_r , Δ_g and Δ_b . That strategy could speed up the interpolation process.



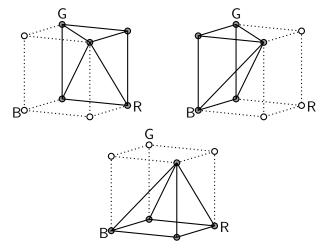






$$V(r,g,b) = V(R_0, G_0, B_0) \frac{V_{000}}{V_{Prism1}} + V(R_0, G_0, B_1) \frac{V_{001}}{V_{Prism1}} + V(R_1, G_0, B_1) \frac{V_{101}}{V_{Prism1}} + V(R_0, G_1, B_1) \frac{V_{011}}{V_{Prism1}} + V(R_1, G_1, B_1) \frac{V_{111}}{V_{Prism1}}$$
(3)

$$\begin{array}{c} \text{Rmk:} \\ V_{\textit{Prism1}} = V_{000} + V_{001} + V_{101} + V_{011} + V_{111} \end{array}$$



The algorithm chooses the pyramid based on the cases:

- if $\Delta_r > \Delta_b$ and $\Delta_g > \Delta_b$, then $p = p_1$.
- ightharpoonup if $\Delta_b > \Delta_r$ and $\Delta_g > \Delta_r$, then $p = p_2$.
- ightharpoonup else $p=p_3$.

3D LUT interpolation: Pyramid interpolation We use the matrix notation and define the solution or the three pyramids (p_1, p_2, p_3) :

$$oldsymbol{V} = egin{bmatrix} V(R_0, G_0, B_0) \ V(R_0, G_1, B_0) \ V(R_1, G_0, B_0) \ V(R_1, G_1, B_0) \ V(R_0, G_0, B_1) \ V(R_0, G_1, B_1) \ V(R_1, G_0, B_1) \ V(R_1, G_0, B_1) \end{bmatrix}$$

$$egin{aligned} oldsymbol{\Delta}_{
ho_1} &= \begin{bmatrix} 1 & \Delta_b & \Delta_r & \Delta_g & \Delta_r \Delta_g \end{bmatrix}^T \ oldsymbol{\Delta}_{
ho_2} &= \begin{bmatrix} 1 & \Delta_b & \Delta_r & \Delta_g & \Delta_g \Delta_b \end{bmatrix}^T \ oldsymbol{\Delta}_{
ho_3} &= \begin{bmatrix} 1 & \Delta_b & \Delta_r & \Delta_g & \Delta_b \Delta_r \end{bmatrix}^T \end{aligned}$$

And by defining the three following matrices:

$$m{C_2} = egin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \ 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

3D LUT interpolation: Pyramid interpolation

Now we can find the interpolation for each pyramid as

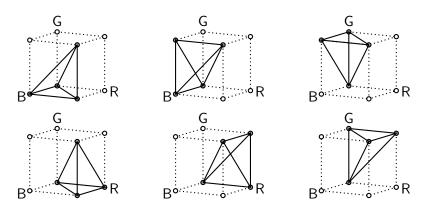
$$V(r,g,b)_{p_1} = \boldsymbol{\Delta}_{p_1}^T \boldsymbol{C_1} \boldsymbol{V}$$

$$V(r,g,b)_{p_2} = \boldsymbol{\Delta}_{p_2}^T \boldsymbol{C_2} \boldsymbol{V}$$

$$V(r,g,b)_{p_3} = \boldsymbol{\Delta}_{p_3}^T \boldsymbol{C_3} \boldsymbol{V}$$

Again, the vectors C_1V , C_2V and C_3V can be computed and stored in advance since they don't depend on the coordinates (r,g,b) or Δ_r , Δ_g and Δ_b . That strategy could speed up the interpolation process.

3D LUT interpolation: Tetrahedral interpolation



3D LUT interpolation: Tetrahedral interpolation

The algorithm chooses the tetrahedron based on the cases:

- if $\Delta_b > \Delta_r > \Delta_g$, we chose the first tetrahedron (t1),
- lacktriangledown if $\Delta_b > \Delta_g > \Delta_r$, we chose the second tetrahedron (t2),
- if $\Delta_g > \Delta_b > \Delta_r$, we chose the third tetrahedron (t3),
- if $\Delta_r > \Delta_b > \Delta_g$, we chose the fourth tetrahedron (t4),
- if $\Delta_r > \Delta_g > \Delta_b$, we chose the fifth tetrahedron (t5),
- ightharpoonup else we chose the sixth tetrahedron (t6).

3D LUT interpolation: Tetrahedral interpolation We use the matrix notation:

$$\boldsymbol{V} = \begin{bmatrix} V(R_0, G_0, B_0) \\ V(R_0, G_1, B_0) \\ V(R_1, G_0, B_0) \\ V(R_1, G_0, B_0) \\ V(R_0, G_0, B_1) \\ V(R_0, G_1, B_1) \\ V(R_1, G_0, B_1) \\ V(R_1, G_1, B_1) \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \Delta_b & \Delta_r & \Delta_g \end{bmatrix}^T$$

3D LUT interpolation: Terahedral interpolation

And by defining the six following matrices:

$$\boldsymbol{T_2} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$$ag{7}_3 = egin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

3D LUT interpolation: Terahedral interpolation

$$\mathbf{T_4} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix} \\
\mathbf{T_5} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

3D LUT interpolation: Tetrahedral interpolation

Now we can find the interpolation for each tetrahedron as

$$V(r,g,b)_{t_1} = \boldsymbol{\Delta}_t^T \boldsymbol{T_1} \boldsymbol{V} \qquad V(r,g,b)_{t_2} = \boldsymbol{\Delta}_t^T \boldsymbol{T_2} \boldsymbol{V}$$

$$V(r,g,b)_{t_3} = \boldsymbol{\Delta}_t^T \boldsymbol{T_3} \boldsymbol{V} \qquad V(r,g,b)_{t_4} = \boldsymbol{\Delta}_t^T \boldsymbol{T_4} \boldsymbol{V}$$

$$V(r,g,b)_{t_5} = \boldsymbol{\Delta}_t^T \boldsymbol{T_5} \boldsymbol{V} \qquad V(r,g,b)_{t_6} = \boldsymbol{\Delta}_t^T \boldsymbol{T_6} \boldsymbol{V}$$

Again, the vectors T_iV , for i=1,2,3,4,5,6, can be computed and stored in advance since they don't depend on the coordinates (r,g,b) or Δ_r , Δ_g and Δ_b . That strategy could speed up the interpolation process.

3D LUT interpolation: comparison of computational cost

	Comparisons	Multiplications	Additions	Storage*
Trilinear	0	7	7	12
Prism	1	5	6	9
Pyramidal	2	4	4	5
Tetrahedron	2.5	3	3	13

^{*} pre-computed coefficient stored at each node

3D LUT interpolation: other approaches

Other ways to interpolate the 3D LUT:

- ► Cellular Regression
- ► Non-uniform lattice 3D LUT
- ► Alternative color spaces

3D LUT interpolation: Cellular Regression

A combination of three-dimensional interpolation and cellular regression.

We apply regression to a small lattice cell (versus the whole cube).

Pros:

- No need to find the position of the interpolation within the cube.
- No need for uniform packing: new 3D structures, like hexahedra, can be used.

Cons:

- ► Higher computational cost
- ▶ Why not use a larger 3D LUT then?

3D LUT interpolation: Non-uniform lattice 3D LUT

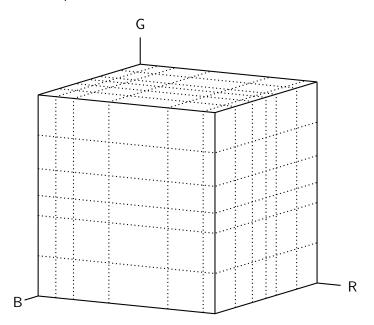
Pros:

- ▶ Perception non-uniformity: We don't see difference between two colors the same depending on where we are in the cube.
- ▶ Does not require additional computational cost (most hardware feature a front 3x 1D LUT)

Cons:

Not 'hardware friendly'

3D LUT interpolation: Non-uniform lattice 3D LUT



Acknowledgment

- ► C. Poynton, Digital Video and HDTV Algorithm and Interfaces, Morgan Kaufmann, San Francisco, 2003.
- ► Henry R. Kang, Computational Color Technology, SPIE Press Book, 17 May 2006.